

Ancient standards of volume: negevite Iron Age pottery (Israel) as a case study in 3D modeling

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Abstract

Hand-made cylindrical vessels unearthed in excavations of Iron Age IIA sites in the Negev Highlands constitute the largest and most dominant ceramic assemblage of simple-shaped vessels found in Israel. The volumes and linear dimensions of these vessels were analyzed based on computer 3D models reconstructed according to their drawn profiles. This analysis revealed the rules that could have been employed by the ancient potters in order to produce vessels of given volumes. These rules demonstrate the human ability to reveal approximate (but inherent) geometric relationships between form and volume and deploy them in everyday life.

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1. Measurement of volume in the ancient near east

Could people in antiquity estimate volumes in a quick and robust way, without applying complex formulae or having to pour liquids and solids into pre-measured containers? Could they make containers with pre-planned volumes? A logical way to check this is to investigate the size and volume of pottery vessels – the most common containers in ancient times – in accordance with measurement units and procedures known from ancient texts.

Archaeology and ancient texts provide a large body of data regarding units of measurement, the ways in which ancient people took measurements, and the measurement tools they used. Numerous finds related to measuring systems have been unearthed in ancient Egypt and Mesopotamia – wood, stone, and copper rods, inscribed weights, and measuring vessels. The textual evidence can be found in invoice ostraca, mathematical papyri, numerous cuneiforms tablets devoted to mathematical rules in everyday activities [9,17] and the Bible [15].

Methods of measurement can be divided into two branches – measurements of length and weight and measurements of area and volume. The former, demanding a single-dimensional characterization and comparison, can be traced to very ancient times: weights and rulers similar to those we use today have been discovered in a number of excavations – for example, a collection of wood rods and measuring pots from ancient Egypt, now in the Petrie Museum (Institute of Archaeology, University College London), the Nippur rod, and the “cubit of Gudea” – a graduated ruler on the statue of Gudea, prince of Lagash, dated to the late third millennium BCE [18].

Intuitively, the measurements of areas and volumes are less straightforward. Ambiguity in “eye estimates” demands definite and common *rules* of measurement. An example of such a rule attributed it to the ancient Greeks, which remains practical today, is the algorithm of area measurement expressed by superimposing a square grid of cells, and calculating the number of these cells within the area.

This paper focuses on measuring volumes of ancient containers – a basic issue in daily wholesale and retail activities. This can be done either manually (e.g., by pouring water into them) or digitally, with the help of CAD software. This article

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deals with the latter. Our goal is to present both the methodology and a case study.

According to Mesopotamian cuneiform tablets and Egyptian papyri, the standard algorithms for measuring volumes dealt with cylinders [11,24] and the basic rule did not differ from the modern one:

$$V = SH = \pi(D^2/4)H \quad (1)$$

where V is the volume of the cylinder, S the area of its base, D the diameter of its base and H the cylinder's height. Interpreting Egyptian papyri, the value of π employed was either 3, or 3.16 [22].

Mathematically equivalent formula, with the basis circumference A replacing the diameter D , and π taken as 3, is reported to have been employed by the Assyrians [25]:

$$V = A^2/12H \quad (2)$$

The most complex bodies discussed in Egyptian texts are pyramids and cones, both complete and truncated [24]. As above, the formula applied for calculating their volume reflects the modern equation:

$$V = (1/3)SH \quad (3)$$

However, the description of the algorithm, especially in the case of truncated bodies, often looks as if it is aimed at imitating Formula (3), or a formula for truncated pyramid/cone without understanding its modern logic [22,24].

Compared to ancient Egypt, Mesopotamia and Babylonia, the knowledge on the measurement systems in ancient Israel is limited. The Hebrew Bible (Exodus 16: 18, 29: 40, Leviticus 6: 20, Numbers 28: 5, Isaiah 5: 10), late-Iron II inscribed weights [14,19], and late-Iron II ostraca mentioning definite quantities of wine, oil and grain [1] all refer to a local system. This system had much in common with those of the neighboring cultures, but it employed local measures of volume – *bath*, *ephah*, *issaron* and *omer* (the Hebrew word *issaron* probably means “the tenth part”).

What are the volume-values of these biblical units? How was the volume of a vessel estimated? To what extent was precision in volume estimates important, and what error of measurement was tolerated? Was the daily activity of potters, merchants, and consumers based on formulae of the Eqs. (1)–(3)? Is it possible that mathematical rules – in the case of both the great empires and ancient Israel – were known to the intellectual elite while the daily practice was based on eye estimates or some approximate rules that did not include multiplication and the use of π ? And if positive, what are these approximate rules?

This paper aims at investigating some of these questions on the basis of Iron Age IIA hand-made pottery vessels found in excavations in Negev Highlands sites [4,6,7,16].¹ We have

¹ There are no stratigraphically related, short-lived, reliable radiocarbon dates for the Iron IIA in southern Israel. In the north, short-lived radiocarbon samples have dated this phase to ca. 980–840/30 or to ca. 920 to 840/830. The absolute date of the Negev Highlands sites (which belong to the *Early* Iron IIA) is debated: 10th century BCE according to the traditional chronology; late 10th-to-early 9th century according to the low chronology [15].

chosen this case because the assemblage is characterized by a large number of easy to measure cylindrical vessels – the largest and most dominant Bronze or Iron Age assemblage with simple shapes found in Israel. As an additional source of information we examined vessels from the late-Iron II (late 8th and 7th century BCE) site of Tell el-Kheleifeh, located further south, at the head of the Gulf of Aqaba [20].

2. The Negevite pottery

The Negev Highlands is a remote, isolated area of the arid zone of southern Israel. The major ancient desert roads bypassed it on the east and north. Yet, thanks to the higher elevation (400–1000 m above sea level) annual rainfall is more significant (100–200 mm per year) than in the surrounding areas and run-off water can be directed to the *wadis* (dry riverbeds), which may be used for seasonal agriculture if embedded with enough soil. Indeed, this improved ecological niche features evidence for human activity in several periods in antiquity – activity which was based on pastoral nomadism accompanied by seasonal dry farming. The occupational history of the area is characterized by sharp oscillations: several short waves of relatively strong activity versus long periods with no remains at all.

Almost 400 Iron IIA sites have been surveyed in this region (as opposed to no Iron I and almost no late-Iron II sites). They comprised enclosed settlements, small farms, groups of dispersed houses and various installations [7,13]. Scholars debated the nature of the enclosed settlements – forts built by Israelite monarchs, e.g., [5], or habitation places of sedentarizing nomads [10] – and the conditions that enabled their existence in this arid zone: more humid climate than that which prevails in the region today [23] or socio-economic changes that were induced by the neighboring, sedentary cultures [10].

Though the origin of the Iron IIA Negev Highlands people remains unclear, exchange with the outer world is evident in the ceramic record. It is therefore probable that the system of volume units used for the production of pottery vessels reflects knowledge of methods used in neighboring, more developed regions.

The pottery found in the Iron IIA Negev Highlands sites can be divided into two groups: a somewhat limited number of wheel-made vessels which were brought from the sedentary lands to the north, and the southern, hand-made cylindrical vessels, known as the “Negevite ware” [4,6,7,16,20]. In this paper, we focus on the latter group. The cylindrical shape of the Negevite vessels is the result of the coiling process [21]: the potter made a round base and then added clay strips – one on top of the other – on its circumference. Traces of firing on many of the vessel indicate that they were used for cooking, though other functions cannot be excluded. The fact that this pottery is absent from sites in the neighboring sedentary centers means that transportation of Negev products was probably done in northern, wheel-made storage-jars.

The assemblage under investigation includes 34 vessels [4,6,7,16]. In order to enhance the reliability of the study,

we added 13 vessels from the late-Iron II site of Tell el-Kheleifeh, located further south, at the head of the Gulf of Aqaba [20]. Admittedly, this site is a very different from those in the Negev Highlands: it seems to have been an Assyrian fort; it dates two centuries later than the Negev Highlands sites; and it is located on a major commercial artery. Still, the cylindrical Negevite vessels found there are similar in shape, ware, and size-range to those unearthed at the Negev Highlands sites.

3. Methodology – 3D reconstruction of vessels

Three-dimensional reconstruction of ceramic vessels provides the background for our study. The 3D models presented in this paper were constructed with the *Rhinoceros*TM 3D-modeling software (<http://www.rhino3d.com/>). Fig. 1 demonstrates the stages of the process in the case of a Negevite vessel from Har Boqer, stored in the Tel Aviv University study collection. To construct a model, a vector drawing of the vessel's profile was built based on its manual drawing imported by the *Rhinoceros*TM. The vertical symmetry axis of the profile was built next, and the 3D model of the vessel was obtained by revolving the profile around this axis (for a detailed description of the process see [26]). Note that the Har Boqer vessel (Fig. 1a) is the only one which was physically available to us; for the other vessels the process began with the manually drawn profiles (e.g., Fig. 1b).

Below we investigate the regularities in the vessels' form and volume. To estimate the linear dimensions of a vessel, one can employ both inner and outer measurements. We begin with the inner measurements, which directly define the volume. We then move to the outer ones, assuming it was more convenient to take them when constructing the vessel.

The calculation of the volume of the 3D model by the *Rhinoceros*TM software does not employ formulae of the Eqs. (1)–(3), which are based on diameter and height. Rather, it is based on the exact geometry of the 3D model (Fig. 1d, e), and is provided by the software automatically.

Regardless of the form of a given vessel, the software estimate of its volume agrees to a great degree with the manual measurement [26]. As a test of the accuracy, the volume of the vessel in Fig. 1a was measured by two methods: manually by pouring grain into it and numerically based on its 3D model (Fig. 1d) [26]. The results were almost identical: 3.59 and 3.60 L, respectively.² Therefore, in what follows we consider the software estimates of the 3D models' volume as adequate estimates of the volume of the real vessels.

The vessels whose shapes are non-cylindrical cannot be characterized by D and H only, and are therefore excluded from further analysis (Fig. 2). We measured deviation of the vessels' shape from the cylinder in two ways:

First, we estimated the relative difference ΔV between the vessel's volume V_{software} as calculated by the software, and the volume V_{cylinder} as calculated according to Formula (1):

$$\Delta V = |V_{\text{software}} - V_{\text{cylinder}}| \quad (4)$$

We then excluded from the analysis six vessels, for which $\Delta V > 0.4$ L. Note that in Formula (4) we employ the absolute rather than relative difference between the volumes, because one of our goals is to identify the units of volume used in the Negev Highlands.

Second, we excluded three more vessels whose base is arched (Fig. 2c); they do not fit our understanding of the coiling process and cannot be characterized by D and H either.

We did not exclude from the analysis a few vessels whose vertical profile is a trapezoid; their volume can be properly estimated on the basis of D and H (see Fig. 1c).

The 3D models of the 38 vessels included in the analysis – 27 from the Negev Highlands and 11 from Tell el-Kheleifeh – are presented in Fig. 3. Their outer linear dimensions and volumes as estimated by the *Rhinoceros*TM software are given in the Appendix.

4. Analysis of the vessels' linear dimensions and volume

4.1. Units

Evidently, it is awkward to use modern units of centimeters and liters for ancient vessels. We should therefore turn to systems of units which were “natural” for the Negevite potters.

4.1.1. Length

Numerous pieces of evidence attest to the widespread use of Egyptian units of length in ancient Israel [19], hence in what follows we apply them to the Negevite vessels. The basic unit of this system is the *royal cubit*, reported in different sources to equal 52.3–52.5 cm [12,19]. One cubit consisted of 28 *fingers* (f), that is, $1 f = 1.865\text{--}1.875$ cm. The *short cubit*, which is equal to 24 fingers, was used less frequently [12,19]. A royal cubit was often considered as consisting of seven *palms*, four fingers each. The 1/4th of the royal cubit, which equals seven fingers, was also widely used [12,19].

4.1.2. Volume

Regarding volume, it is widely accepted that the basic units in ancient Israel were *bath* and *ephah* [19]. According to the Hebrew Bible, these units denote the same volume – *bath* was used for liquids and *ephah* for dry products: “The ephah and the bath shall be of the same measure, the bath containing one tenth of a homer, and the ephah one tenth of a homer; the homer shall be the standard measure” (Ezekiel 45: 11). The 10th of a *bath* – *issaron* – and the 10th of an *ephah* – *omer* – were employed for measuring smaller volumes [19] and are also mentioned in the Bible: “An omer is the tenth part of an ephah” (Exodus 16: 36). According to (Exodus 16:18), one *omer* (equal to one *issaron*) makes a daily portion of food.

² In another paper one of us compared the accuracy of the 3D models volume estimates to manual measurement (by pouring water) for 15 symmetric wheel-made vessels whose volume vary between ca. 1 and 40 L [26]. In all cases the difference remained below 1–3%. The more significant deviations were caused by imprecise profile drawing.

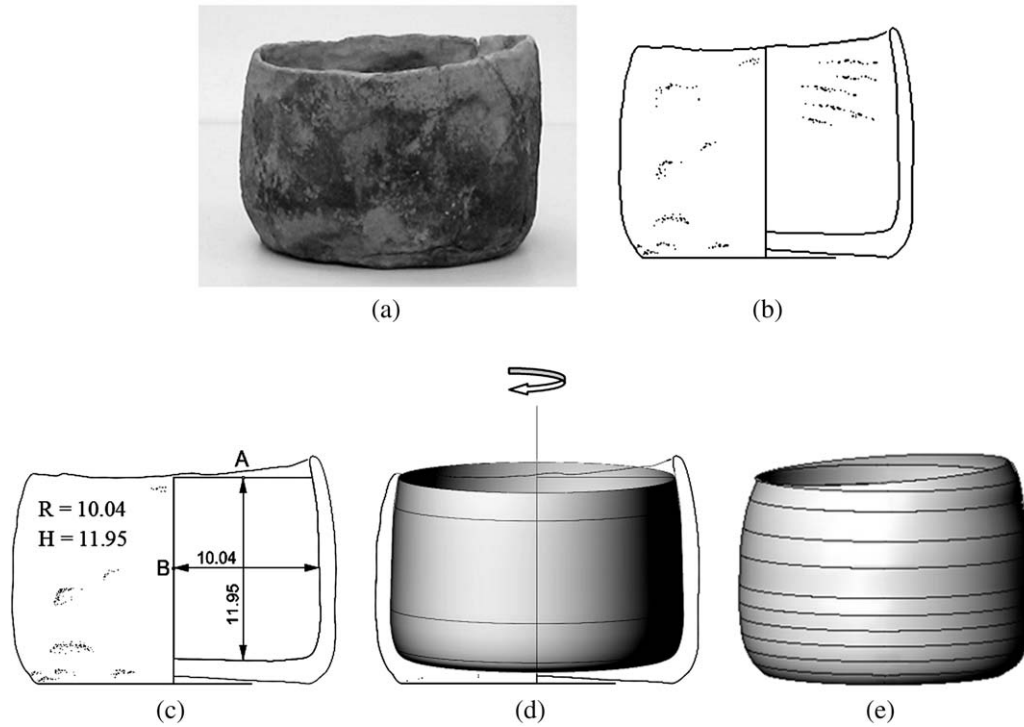


Fig. 1. The stages of the 3D-model construction of a vessel from Har Boqer (Tel Aviv University study collection). (a) An image of the vessel; (b) the vessel’s manually-drawn profile; (c) vector drawing with measurement according to the inner profile; (d) the vessel’s 3D model, with the rotation axis marked and the inner side shaded; (e) the 3D model shown at the same angle as the vessel in (a), with the outer side shaded.

The precise value of these units is not known. According to different sources, *bath* and *ephah* vary between 20 and 24 L [19], i.e., *issaron* and *omer* being 2.0–2.4 L. A jug found in Stratum II (8th century BCE) at Tel Beer-Sheba carries an inscription “Half-(measure) of the King” [2]. Its capacity – 1.2 L – supports the estimate of *issaron* as close to 2.4 L.

In what follows we base our discussion on the *bath* and the *issaron*, keeping in mind that the same statements are valid for *ephah* and *omer*. We begin the presentation of the results with centimeters and liters and then switch to *ingers* (f) and *issarons* (s).

4.2. The distribution of the vessels’ volume

Fig. 4 presents the volume distribution of the 38 cylindrical vessels presented in Fig. 3, in a resolution of 0.2 L. Three modal groups, corresponding to the 1.6–3.0, 4.4–4.8 and 6.4–7.6 L intervals can be visually traced. In what follows

we accept this partition, acknowledging that the sample size is below the level necessary for statistical inference.

In what follows, we take 3.2 L as an upper boundary of the first group; the volumes of the 21 vessels within this group vary between 1.54 and 3.10 L.

The mean volume of the vessels in this group is 2.26 ± 0.091 L, and the median volume is 2.25 L. These values agree to a great degree with the aforementioned estimates of the *issaron* as 2.0–2.4 L, and we thus assume that the *issaron* was the Negev’s unit of volume.

The estimate of the *issaron* as being close to 2.3 L is supported by the following numeric considerations: (1) its value can be related to a volume of a cube with the edge of 1/4th of a royal cubit = 7 f. The volume of such a cube equals $(7 f)^3 = 343 f^3$, and depending on the numeric value of a finger, results in 2.235–2.281 L – 2.26 L at average. (2) the shape of the aforementioned 1.2 L jug from Tel Beer-Sheba is close to a sphere 7 f in diameter; its volume is thus close

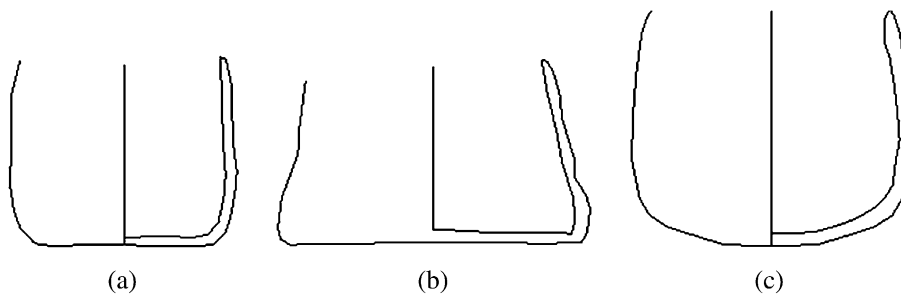


Fig. 2. Typical profiles of the vessels included in (a) and excluded from (b and c) this study.

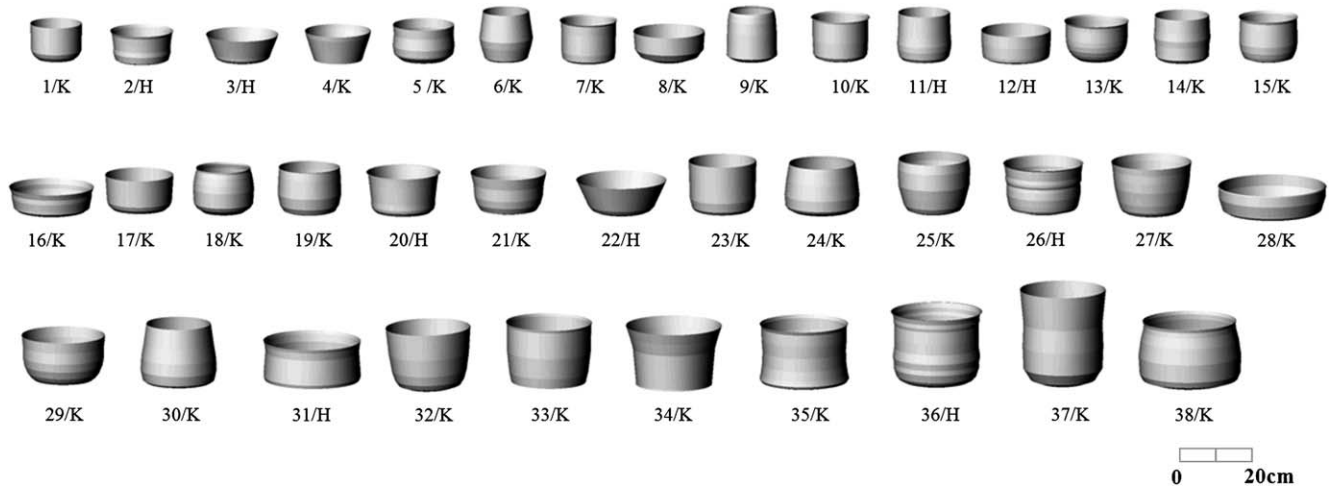


Fig. 3. 3D models of the 38 vessels studied in order of increasing volume; see data on the vessels' linear dimensions and volume in the Appendix (H = Negev Highlands; K = Tell el-Kheleifeh).

to: $V_S = 4\pi(7f)^3/3$. Employing the two Egyptian approximations of $\pi - 3.0$ and 3.16 – one obtains V_S as either $343 f^3/2$ or $359 f^3/2$. The former is exactly half of the volume of a $7 f$ -edge cube, while the latter results in an *issaron* of 2.34 – 2.37 L.

Altogether, four of five sources of estimates for the *issaron* lay within 2.25 – 2.26 L: mean and median of the first modal class, volume of a cube of a $7 f$ edge, and twice the volume of a sphere of the $7 f$ diameter for $\pi = 3$. One of the estimates – twice the volume of a $7 f$ diameter sphere for $\pi = 3.16$ – equals to ~ 2.36 L. We therefore argue that for the Negevite vessels 1 *issaron* equals the mean volume of the vessels of the first modal class – 2.26 L (and, thus *bath* equals ~ 22.6 L). To translate the volumes from cubic fingers to *issarons*: $2.26 \text{ L} = 351.7 f^3$. In what follows we are using these precise numbers for the sake of convenience; the results will not change if we take *issaron* to equal, say, 2.2 or 2.3 L. However, we do consider the above results as sufficient to narrow the value of *bath* to the interval 22 – 23 L.

Fig. 5 presents the distribution of the 38 Negevite vessels by volume measured in *issarons*; one can easily distinguish three modal classes around the volumes of one, two, and three *issarons*. We consider this as an additional confirmation of the use of *issaron* as the unit of volume in the Negev.

Let us investigate the relation between the linear dimensions and volume of the vessels of the first modal class in depth.

4.3. Linear dimensions and volume for vessels of the first modal class

Table 1 presents the statistics of the inner and outer diameter, height and volume of the 21 Negevite vessels of the 1.54 – 3.10 L (or 0.68 – 1.37 *issarons*) class.

The variation in D and H is of the same order as that of the volume, and this shows that the vessels' diameter and height

are dependent – if not, then the CV_V of the volume will be equal, approximately, to:

$$CV_V \approx (4/\pi) \sqrt{[2CV_D^2 + CV_H^2]} = 33.8\% \quad (5)$$

where CV_D and CV_H are the CV of the inner diameter and height. The result in Eq. (5) is almost two times higher than the one in the table. Fig. 6 demonstrates this dependency for inner and outer measurements (in both cases $r^2 = 0.37$, $p < 0.01$). Note that, exclusion of the extreme values results in the essential growth of r^2 . For example, excluding the minimal and maximal vessels of 1.54 and of 3.10 L (both might be wrongly classified as belonging to this class), results in $r^2 = 0.54$ for the inner and outer measurements.³

When switching from inner to outer measurements, the difference between outer and inner dimensions is $\approx 0.5 f$ for H and $\approx 1.1 f$ for D (Table 1). There are no correlations between the width of the side, the width of the bottom and the volume of a vessel – each of the three values of the r^2 is below 0.1 , that is, we can use the formula:

$$V = SH = \pi(D - 1.1)^2/4(H - 0.5) \quad (6)$$

for calculating the volume of a cylindrical vessel, based on its outer measurements.

The clustering of the vessels around 2.26 L, and the fit between this value and the ancient *issaron*, causes us to assume that the Negevite potters aimed at one *issaron* when producing a vessel that belongs to the first modal group. The negative correlation between D and H means that if the base of a vessel was bigger, the potters made it shorter; if the base was smaller,

³ The variation in the vessels' linear dimensions in Table 1 is twice higher than that estimated for a set of fine-ware bowls discovered in Tell Leilan, Syria (ca. 2300 B.C.) and similar to that estimated for another group of fine-ware bowls discovered there [3]. It is noteworthy that human ability of eye estimation of length results in $CV \sim 2\%$; the variation in linear dimension of the professionally made pottery as revealed in ethnoarchaeological studies is about 4% [8].

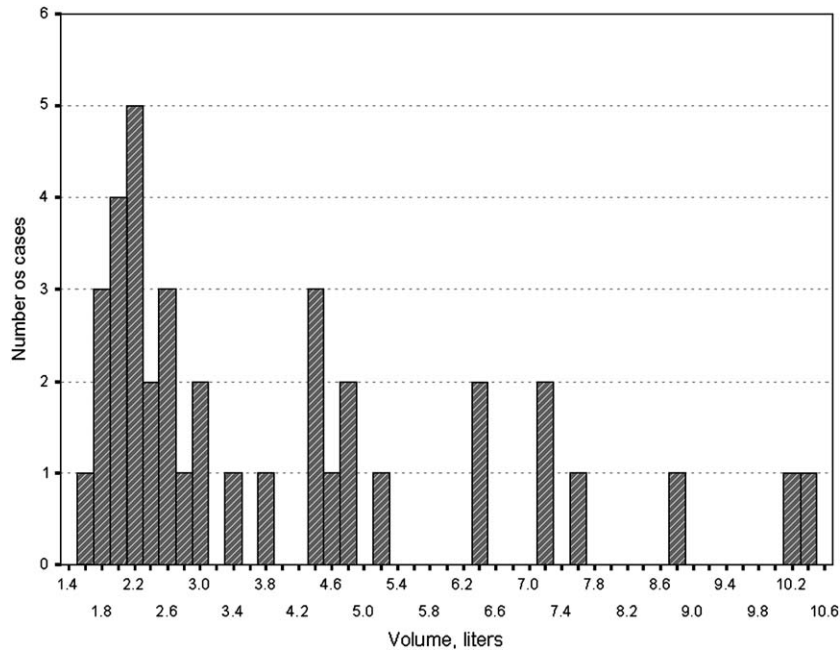


Fig. 4. Volume distribution of the 38 Negevite cylindrical vessels.

they produced a higher vessel. Does this mean that the potters applied the inverse of Formula (1) in order to decide what should be the height of a vessel after they established the base? Our hypothesis is that they used a much simpler rule.

5. The “Palm Rule” and its verification

5.1. The implications of the unit system’s logic

Clustering the vessels around integer values of the volume — expressed in *issarons* — has ‘numeric’ implications that relate

between their linear dimensions and volume (Fig. 7). The curves in the figure represent the domains of the outer (D, H) dimensions, for which the volume of the cylinder, calculated according to Formula (1), equals to 1 ± 0.2 , 2 ± 0.2 , and 3 ± 0.2 *issarons* (20%-width of the corridor is chosen arbitrarily). The straight lines represent three linear relationships between the cylinder’s diameter and height: $D + H = 16$ f, $D + H = 20$ f, and $D + H = 24$ f.

As can be seen in Fig. 7, the corridors of 1 ± 0.2 and 2 ± 0.2 *issarons* contain lengthy sections of the corresponding lines. The meaning of this intersection is that for the vessels of

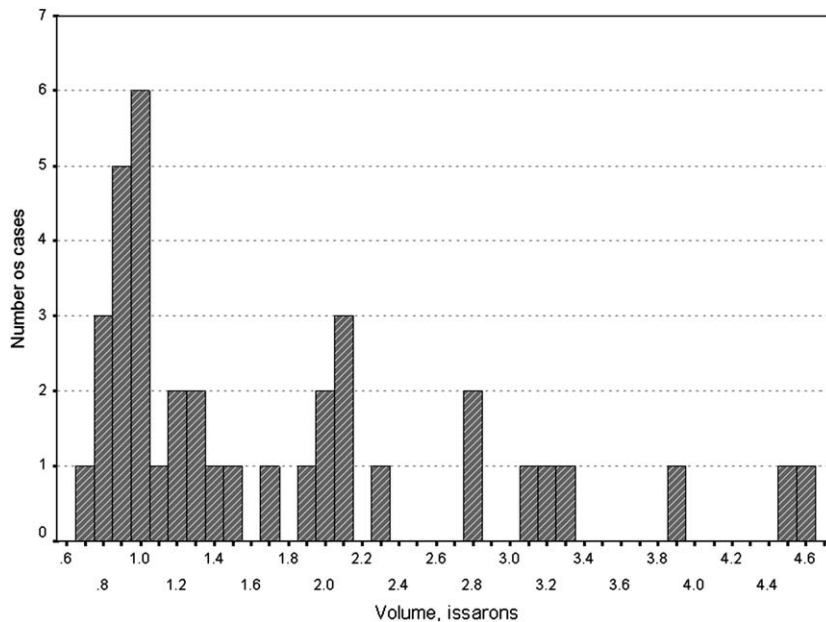


Fig. 5. Distribution of the Negevite vessels’ volume as expressed in *issarons*.

Table 1
Statistics of the 21 Negevite vessels of the 0.68–1.37 *issarons* group

| Dimension | Min | Max | Mean | STD | CV ^a (%) |
|-------------------------|------|-------|-------|------|---------------------|
| Inner diameter (finger) | 6.84 | 11.63 | 8.91 | 1.14 | 12.8 |
| Inner height (finger) | 3.57 | 7.24 | 5.71 | 1.11 | 19.4 |
| Outer diameter (finger) | 8.02 | 12.44 | 10.00 | 1.12 | 11.2 |
| Outer height (finger) | 3.83 | 7.91 | 6.22 | 1.12 | 18.0 |
| Volume (issaron) | 0.68 | 1.37 | 1.00 | 0.19 | 19.0 |

^a CV – coefficient of variation equals STD/Mean.

1 ± 0.2 *issarons*, the sum of D and H , when expressed in fingers, will be close to 16 f as far as their $D:H$ proportions remain within the interval 8:8–3:13. Respectively, for the vessels of 2 ± 0.2 *issarons* the sum of D and H will be close to 20 f as far as their $D:H$ proportions remain within the interval 9:11 – 4:16. Note that both intervals of the $D:H$ contain all vessels of the “practical” proportions. For the next size, of 3 ± 0.2 *issarons*, the straight line does not remain within the corridor, but the deviation is still reasonable. For larger vessels the correspondence does not hold (Fig. 7).

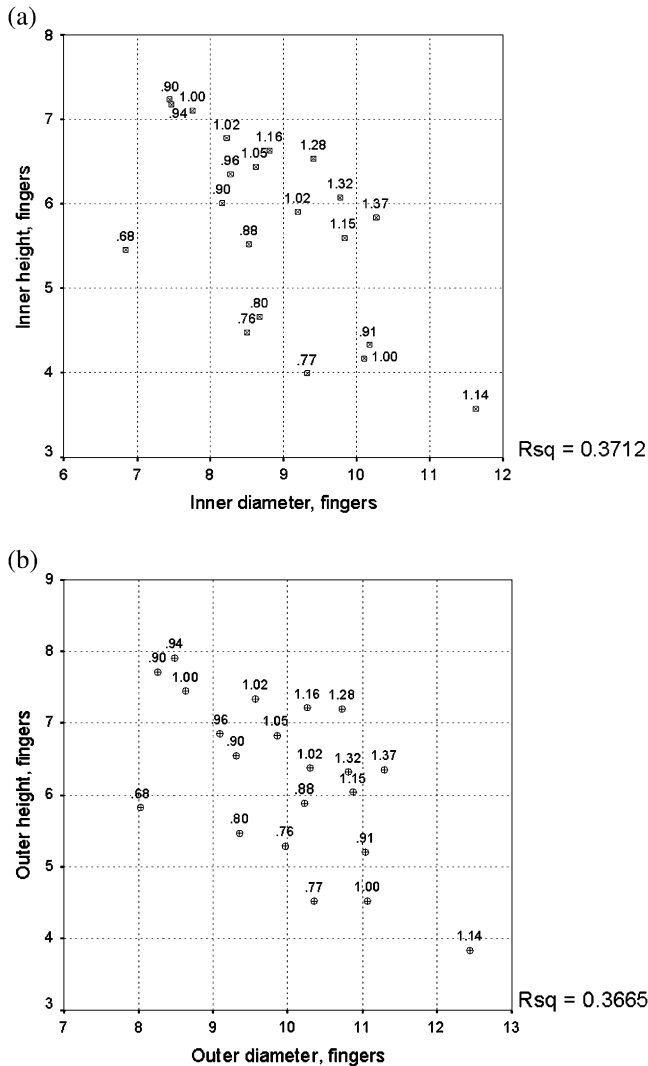


Fig. 6. Relationship between height and base diameter (both measured in fingers) in the 21 Negevite vessels of the first modal class: (a) inner, (b) outer measurement.

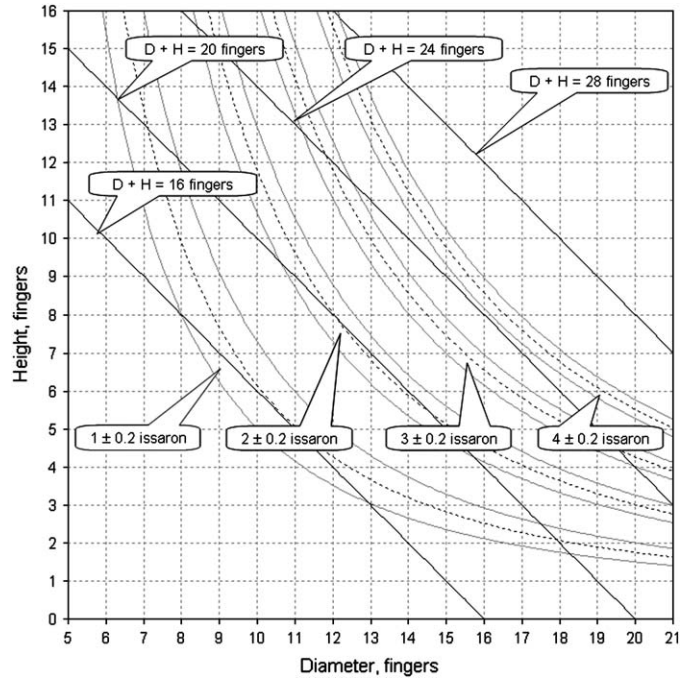


Fig. 7. The (D, H) corridors corresponding to the vessels equal to 1 ± 0.2 , 2 ± 0.2 , 3 ± 0.2 and 4 ± 0.2 *issarons*, with the straight lines satisfying the conditions $D + H = 16 f$, $D + H = 20 f$, $D + H = 24 f$, and $D + H = 28 f$ superimposed.

The above relationships are inherent for the finger-issaron system of units, and can be turned over, that is, the potter could know the values of $D + H$ that are characteristic of the vessels of 1, 2 and 3 *issarons*. Aiming at cylindrical vessel of, say, one *issaron*, the potter could have decided about its proportion, make the base of a given diameter and then add the coils until the side reaches $16 f - D$. The same approach can be applied for constructing the vessels of two *issarons*, and, with the possibility of being mistaken for the proportions close to $D:H = 8:16$, also for the vessels of three *issarons*, but not for bigger vessels.

It is impossible to tell whether the potters indeed deployed the above relationship (and the observed clustering of the vessels’ volumes is thus the result of its implementation), or whether they followed some other rules and the observed fit is a consequence of the above-mentioned inherent mathematical relationships. In any case, the clustering of the Negevite vessels around the volumes of 1, 2 and 3 *issarons* is equivalent to clustering the values of the outer $D + H$ around the 16 f , 20 f and 24 f (Fig. 8).

An (admittedly weak) argument in favor of the knowledge of the relationship by the potters can be the absence of vessels of precise volumes larger than three *issarons* (Fig. 5). If the volume of the vessels was measured *after* the construction, nothing would have prevented the potter from making larger vessels. But, of course, the larger vessels could simply have been lost.

5.2. The hypothesis

Can we take one of the directions of the causation above? To decide, let us select the vessels of the volume close to 1, 2, and 3 *issarons*, supposing that they better “preserve” the rules of their

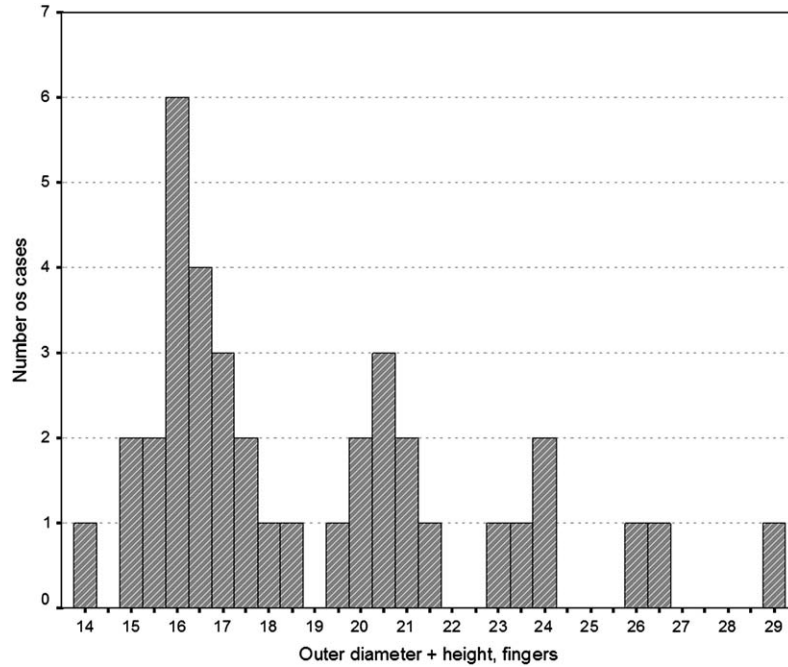


Fig. 8. The distribution of the outer $D + H$ for the 38 Negevite vessels.

construction. Fig. 9 presents 25 of the 38 vessels, the volume of which differ from the nearest integer in 0.2 *issaron* or less.

The figure shows that the values of D and H in the centers of the vessel clouds differ by two fingers. For the three centers, marked by black stars, $D = 10$ f, $H = 6$ f; $D = 12$ f, $H = 8$ f; and $D = 14$ f, $H = 10$ f, respectively. Note that the star and two of the three vessels in the three *issarons* volume category are located beyond the domain. Another observation is that the center of the two *issarons* cloud differs by four fingers in height from the widest vessels, or four fingers in diameter from the highest vessels of one *issaron* category (gray stars).

Our hypothesis is, therefore, that the Negevite potters did know how to build a vessel of one *issaron* based on its linear dimensions: decide about the diameter of the base and raise the side until the sum of its D and H reaches four palms (16 fingers). To build a vessel of two *issarons*, they could use the model of the ‘average’ vessels of one *issaron* and then increase its diameter by half palm (two fingers) and raise its side in an additional half palm. Using the extreme models, they could increase the diameter of the highest vessel or the height of the widest one by one palm. Only the “central” version of the rule is observed in the case of the transition from 2 to 3 *issarons*.

Let us label this hypothesis as the “Palm Rule”.

5.3. The assemblage is limited to vessels satisfying the Palm Rule

Fig. 10 combines the elements shown in Figs. 7 and 9, and presents all 38 vessels as well as the straight lines of $D + H = 16$ f, $D + H = 20$ f and $D + H = 24$ f. The addition of the vessels of the imprecise volume does not change the clustering.

Note, that the $D:H$ proportions of the two *issarons* vessels do not cover the range that satisfies the Palm Rule from

9:12 to 4:16. Yet, the three vessels of 1.49, 2.09 and 2.79 *issarons* (Fig. 10) seem to indicate that this is due to the limited size of the sample.

Note, also, that all linear dimensions participating in the Palm Rule are measured in palms and half-palms, that is, the Palm Rule does not demand precise measurements.

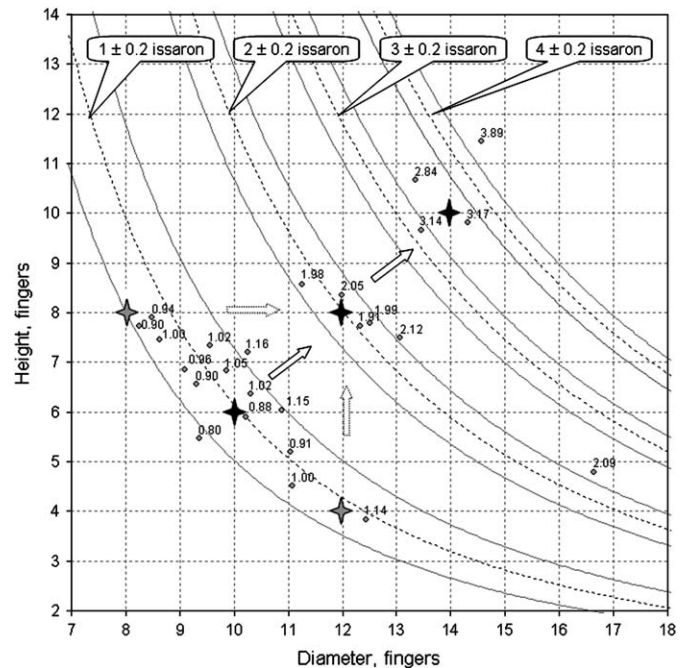


Fig. 9. Domains of (D, H) providing the cylinders of the integer volumes ± 0.2 *issarons* and 25 sample points representing the vessels that differ less than 0.2 *issaron* from the nearest integer volume. Black stars represent the points close to the center of the clouds, where $D + H$ is equal to 16, 20 and 24 fingers. Gray stars represent the boundaries of the one *issaron* category.

5.4. Limitation of the Palm Rule

The Palm Rule cannot be extended to volumes above three *issarons*. Continuing the line connecting the black stars in Fig. 10 towards larger vessels, it can be noted that the progress from three to four *issarons* demands one- (and not two-) finger increases in *D* and in *H*. Besides being a more complex rule, the latter demands that the Negevite potters were able to conduct precise measurements, which we doubt.

6. Conclusions

This study demonstrates the potential of the 3D modeling of ceramic containers and automatic computation of their volume for the study of the past.

Based on the computer 3D models we analyzed the volumes and linear dimensions of the Negevite cylindrical vessels – the largest Iron Age assemblage with simple shapes found in Israel – and proposed the Palm Rule that could be employed by the ancient potters in order to produce vessels of one, two and three *issarons*.

We revealed that the volumes of half of the vessels available are concentrated around the value of 2.2–2.3 L, a value corresponding to estimates of the *issaron*, the 10th of a *bath*. Represented in *issarons*, the volumes of the larger vessels are concentrated around two and three *issarons*, thus supporting the hypothesis that the *issaron* was used by the Negevite potters.

We have then assumed that the Negevite potters knew the diameter and height necessary for the construction of vessels of 1, 2 and 3 *issarons*. In order to reveal the rules used by the potters in order to translate linear dimension into volume, we investigated vessels whose volumes in *issarons* are close to the integer values. This analysis brought us to formulate the Palm Rule.

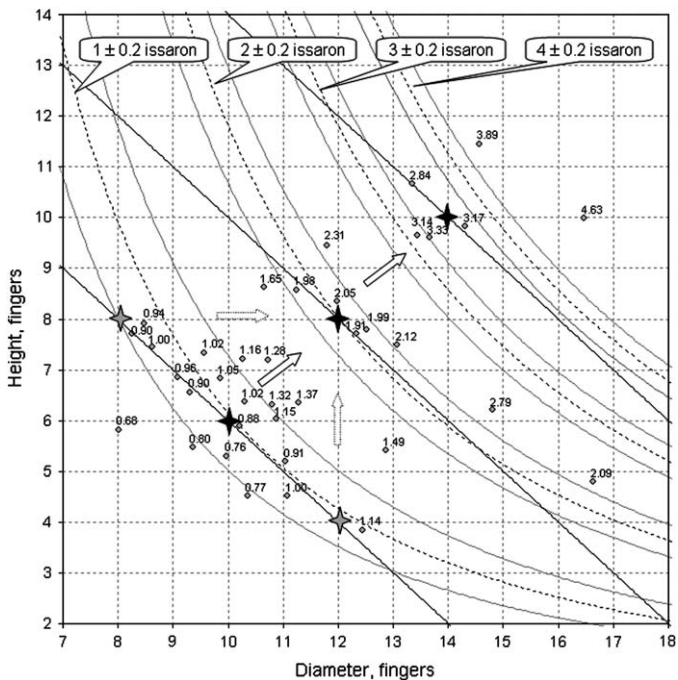


Fig. 10. The Palm Rule shown for all 38 vessels of the assemblage.

Our arguments are all based on the view that the vessels which are larger than one *issaron* are clustered by their diameter and height. Here lies our weak point – the skeptic could argue that the number of these vessels is insufficient for establishing firm conclusions.

To conclude, we argue that the Negev potters knew the relationships between the vessels’ form and volume, and employed this knowledge in everyday life. This hypothesis must not be confined to cylindrical vessels; if true, it should be exposed in vessels of all forms.

We now aim at deploying the huge body of data (drawings and photos) on Bronze and Iron Ages storage vessels for a more comprehensive study of ancient practical computations, especially those related to administration, and their implications for understanding the intellectual capabilities of past people.

Appendix

| Vessel # | Outer base diameter <i>D</i> in cm | Outer height <i>H</i> in cm | Volume <i>V</i> in liters | Outer base diameter <i>D</i> in fingers | Outer height <i>H</i> in fingers | Volume <i>V</i> in <i>issarons</i> | Site ^a |
|----------|------------------------------------|-----------------------------|---------------------------|---|----------------------------------|------------------------------------|-------------------|
| 1 | 15.00 | 10.89 | 1.54 | 8.02 | 5.82 | 0.68 | H |
| 2 | 18.64 | 9.89 | 1.72 | 9.97 | 5.29 | 0.76 | K |
| 3 | 19.36 | 8.45 | 1.74 | 10.35 | 4.52 | 0.77 | K |
| 4 | 17.50 | 10.22 | 1.82 | 9.36 | 5.47 | 0.80 | H |
| 5 | 19.12 | 11.01 | 2.00 | 10.22 | 5.89 | 0.88 | H |
| 6 | 15.44 | 14.43 | 2.03 | 8.26 | 7.72 | 0.90 | H |
| 7 | 17.40 | 12.24 | 2.03 | 9.30 | 6.55 | 0.90 | H |
| 8 | 20.64 | 9.72 | 2.06 | 11.04 | 5.20 | 0.91 | H |
| 9 | 15.86 | 14.79 | 2.12 | 8.48 | 7.91 | 0.94 | H |
| 10 | 17.00 | 12.81 | 2.17 | 9.09 | 6.85 | 0.96 | H |
| 11 | 16.14 | 13.93 | 2.25 | 8.63 | 7.45 | 1.00 | K |
| 12 | 20.70 | 8.46 | 2.27 | 11.07 | 4.52 | 1.00 | K |
| 13 | 19.26 | 11.92 | 2.30 | 10.30 | 6.37 | 1.02 | H |
| 14 | 17.88 | 13.73 | 2.31 | 9.56 | 7.34 | 1.02 | H |
| 15 | 18.44 | 12.77 | 2.37 | 9.86 | 6.83 | 1.05 | H |
| 16 | 23.26 | 7.17 | 2.58 | 12.44 | 3.83 | 1.14 | K |
| 17 | 20.34 | 11.29 | 2.61 | 10.88 | 6.04 | 1.15 | H |
| 18 | 19.18 | 13.49 | 2.63 | 10.26 | 7.21 | 1.16 | H |
| 19 | 20.06 | 13.47 | 2.89 | 10.73 | 7.20 | 1.28 | H |
| 20 | 20.22 | 11.82 | 2.98 | 10.81 | 6.32 | 1.32 | K |
| 21 | 21.12 | 11.88 | 3.10 | 11.29 | 6.35 | 1.37 | H |
| 22 | 24.06 | 10.12 | 3.37 | 12.87 | 5.41 | 1.49 | K |
| 23 | 19.94 | 16.12 | 3.73 | 10.66 | 8.62 | 1.65 | H |
| 24 | 23.06 | 14.43 | 4.31 | 12.33 | 7.72 | 1.91 | H |
| 25 | 21.04 | 16.02 | 4.48 | 11.25 | 8.57 | 1.98 | H |
| 26 | 23.40 | 14.57 | 4.50 | 12.51 | 7.79 | 1.99 | K |
| 27 | 22.42 | 15.61 | 4.64 | 11.99 | 8.35 | 2.05 | H |
| 28 | 31.12 | 8.96 | 4.72 | 16.64 | 4.79 | 2.09 | H |
| 29 | 24.44 | 14.00 | 4.78 | 13.07 | 7.49 | 2.12 | H |
| 30 | 22.06 | 17.66 | 5.23 | 11.80 | 9.44 | 2.31 | H |
| 31 | 27.70 | 11.61 | 6.31 | 14.81 | 6.21 | 2.79 | K |
| 32 | 24.96 | 19.96 | 6.41 | 13.35 | 10.67 | 2.84 | H |
| 33 | 25.16 | 18.04 | 7.10 | 13.45 | 9.65 | 3.14 | H |
| 34 | 26.76 | 18.36 | 7.17 | 14.31 | 9.82 | 3.17 | K |
| 35 | 25.54 | 17.97 | 7.52 | 13.66 | 9.61 | 3.33 | H |
| 36 | 27.24 | 21.39 | 8.78 | 14.57 | 11.44 | 3.89 | K |
| 37 | 26.20 | 28.16 | 10.11 | 14.01 | 15.06 | 4.47 | H |
| 38 | 30.80 | 18.67 | 10.45 | 16.47 | 9.98 | 4.63 | H |

^a H – Negev Highlands; K – Tell el-Kheleifeh.

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