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Computing Abilities in Antiquity: The Royal Judahite Storage Jars as a Case-study

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Abstract Studying the volume of ancient pottery vessels can shed light on the development of complex societies and state apparatus by revealing the means taken to standardize trade and taxation. It can also shed light on the cognitive abilities of ancient people by investigating their knowledge of computing. This paper explores, as a case study, the volume and shape of the *lmlk* ("belonging to the king") royal storage jars, which probably represent the highest level of standardization in eighth century BCE Judah. To estimate the volume of these vessels we constructed a computer 3D model for each jar. The variation in the jars' linear dimensions is about 2–3%, a number that is characteristic of human-made objects produced by professionals without employing measurement tools. Had the potters produced jars of the same height, they could have easily reached 3–4% accuracy in the volume. Surprisingly, the variation in the jars' volume is 7–10%. We hypothesize that rather than height the potters focused on the jars' shape and wall width, estimating the volume according to the jars' outer measurements. We propose a simple way that these measurements could have been taken and suggest a formula that could have been employed by the potters and customers for quickly calculating a jar's volume.

Keywords Royal jars · Administration and standardization in ancient societies · Computing in ancient societies · Biblical units of length and volume

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Introduction

The study of ceramic technology, production, and exchange is an important component of archaeological research (van der Leeuw and Pritchard 1984; Rice 1987; Arnold 1988; Longacre 1991; Sinopoli 1991; Skibo 1992; Christakis 1999, 2005). One important aspect in this domain has only recently attracted scholarly attention. We refer to the study of volume of ancient pottery vessels, which aims at shedding light on the development of complex societies and state apparatus by revealing steps taken in order to standardize trade and taxation (Berg 2004) and at disclosing cognitive abilities of ancient people by revealing their knowledge of computing.

Doumas and Constantinides (1990) studied Bronze Age pithoi found at the site of Akrotiri on the island of Thera. Each of the pithoi carried marks in the shape of concentric rings of varying width. The authors assumed that the number of marks denoted the vessels' capacities in terms of ancient volume units. By dividing the pithoi volume by the number of marks, they estimated this unit to have been ca. 24–26 L. Despite three counter-examples, in which the unit volumes are either ca. 30 L or as low as 14 L (the latter number relates to a smaller vessel), this system of marks can be considered as a step towards standardization, which is especially important in the case of large vessels, when visual estimates of the volume are inaccurate.

Every statistical approach to studying internal order in object measurements begins with an analysis of the samples' histogram. Katsa-Tomara (1990) applied this method to the second set of vessels found at Akrotiri, namely vessels whose volume is below 32 L. These small and average-sized vessels were divided, according to their form, into three groups. The volume histogram for every group is multi-modal, thus demonstrating inherent standards that were employed by the potter. The volumes of 112 bridge-spouted jars of the first group, varying between 0.46 and 15 L, demonstrate clear peaks at a mode close to the maximal size of 15 L and also at modes equal to 1/2, 1/3, 1/6 and 1/20 of the maximum. The modal volumes of the second group, consisting of 17 open-mouth jugs varying between 0.58 and 1.9 L, are either close to maximal, or to 2/3, 1/2 and 1/3 of it. The third group—of 36 ovoid funnel-mouthed vessels (1–32 L)—can also be split into modal groups according to volume: those close to the maximum and those whose volume is 2/3, 1/3, 1/4, 1/8 and 1/16 of the maximum.

Berg's comprehensive discussion of the meaning of standardization (2004) was based on the study of conical Late Bronze Age (Late Cycladic I and II) cups from the islands of Kea and Melos. Berg pointed out the importance of the relative rather than absolute error in the linear dimensions or volume of the vessels. Consequently, she noted that the coefficient of variation (CV), which is calculated as the ratio of standard deviation SD to the mean M, CV = SD/M, reflects the level of standardization better than the SD. From this standpoint, Berg investigated the level of standardization of the Kea and Melos pottery cups for two periods. She compared the CV for the height, rim diameter and base diameter of the cups, and revealed that for the pottery found at Melos the values of CV did not change between the Late Cycladic I and II, while for the cups found at Kea the value of the CV decreased between 2 and 3 times from the Late Cycladic I to the Late Cycladic II, and that this decrease is significant at the 0.01 level. Berg considered alterations in the conic cups' variation as reflecting the changes in socio-economic conditions of pottery production in Kea but not in Melos.

Based on Cuzco excavation data, Costin (1996) noted that standards may depend on the customer of the pottery: the standards sufficient for general production can be lower than those employed for making ceramics for the "emperor".

A comparative study of standardization of modern vessels and ancient vessels of the same form belonging to the medieval Empire of Vijayanagara in south India demonstrated that the variation of the linear dimensions and the volume of the jars produced in the modern workshops is several percent (Sinopoli 1988); this is the level characteristic of all hand-made production (Eerkens 2000; Eerkens and Bettinger 2001). The measurements of archaeological finds in Vijayanagara demonstrated greater variability, but their specific origin and, thus, the sources of variation are impossible to trace.

In a previous work (Zapassky *et al.* 2006), we dealt with the mathematics of volume of simple, cylinder-shaped Iron Age IIA (late tenth–early ninth century BCE) vessels from Negev Highlands sites in southern Israel, believed to have been inhabited by sedentarized pastoral nomads (Finkelstein 1995, pp. 103–126). We based our study on analyzing the height, base diameter and volume histograms. The study revealed several clusters of vessels with the modal volume of 1, 2 or 3 *assirons*, which is one-tenth of the larger *bath*—the basic units of the measurement in ancient Israel (Powell 1992). We also presented the vessels' height and diameter in ancient *palms* (one-seventh of the long cubit), which opened the way to revealing the inherent relationships between width, diameter and volume of these vessels. These relationships entail a simple rule that the potter could follow in order to produce a vessel of a given volume based on the measurements of the vessel's diameter and height during production. We would argue that the use of ancient linear and volume measurement systems is critical for the study of ancient standardization. When expressed in contemporary units, the dimensions and volume of vessels can indicate the human ability to reveal approximate (but inherent) relationships between form and volume and to deploy them in everyday life.

The above results call for an investigation of more complex vessels, which were produced by advanced, bureaucratic societies. In this article, we turn to the kingdom of Judah in the Iron Age IIB (eighth century BCE). At that time, Judah reached full-blown statehood and developed an advanced bureaucratic system that operated under the hegemony of the Assyrian empire (Jamieson-Drake 1991; Finkelstein 1999). One of the manifestations of Judah's bureaucratic apparatus is the late eighth century BCE *lmlk* storage jars (Vaughn 1999; Barkay and Vaughn 2004; Ussishkin 2004; Fig. 1). These vessels, easily identifiable according to both shape and size, are found in destruction layers inflicted in the course of Sennacherib King of Assyria's assault on Judah in 701 BCE, during the reign of King Hezekiah (Ussishkin 1982; Grabbe 2003). Some of their handles carry a seal impression that consists of three components: the legend *lmlk* (in biblical Hebrew "belonging to the king"—hence the description of the vessels as "royal" storage jars); one of two types of emblem (four-winged and two-winged; Fig. 2) widely regarded as royal insignias; and the name of one of four Judahite towns: Hebron, Ziph, Sochoh or the enigmatic *mmst*, still unidentified. These places have been thought to be the location of potters' workshops, royal plantations, or central storage facilities. The volume of the *lmlk*

Fig. 1 *lmlk* storage jar from Lachish; Registration Number 8162/6(2).

storage jars was apparently calculated according to the biblical unit *bath*, which measures ca. 22.5 L; it was found to be close to two *bath*s, that is, ca. 45 L (for the biblical *bath* see Powell 1992).

The *lmlk* legend makes it clear that these storage jars were in the service of the Judahite administration. They were probably used for storing valuable liquids such as wine and oil for military or trade function (Vaughn 1999). Therefore, we assume that it was essential for the producers and consumers to accurately measure their

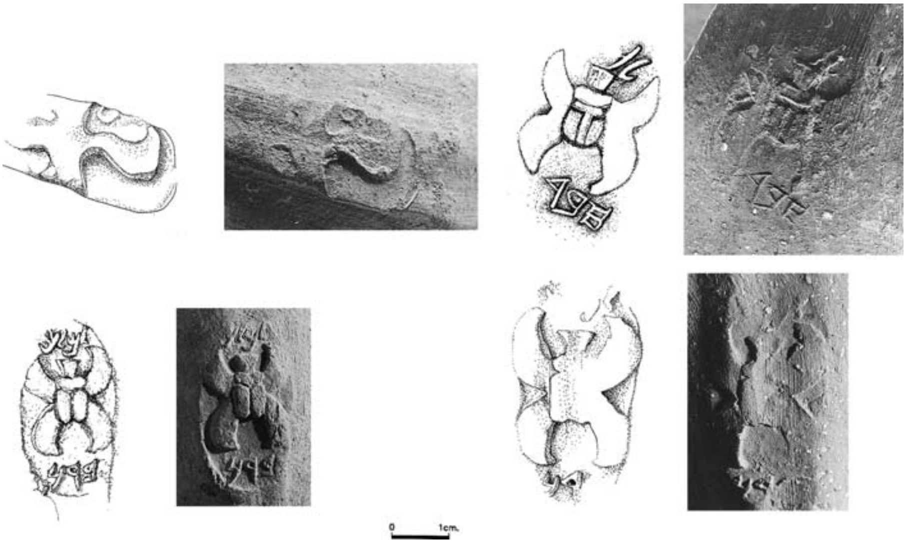

Fig. 2 Two-winged (**a**) and four-winged (**b, c, d**) *lmlk* seal impressions; all from Lachish (Ussishkin 2004).

volume. Contemporary stone weights (Kletter 1998) and biblical references (e.g., Isaiah 5:10; Ezekiel 45:11, 13–15) indicate that commodities were accurately measured in late-monarchic Judah. Hence, contra Ussishkin (2004, p. 2145) it is reasonable to assume that the *lmlk* storage jars represent a high level of standardization in eighth century BCE Judah. Our aim is to check this hypothesis, to investigate the Judahite state administration's level of standardization and precision, and to study the people of Judah's capacity for accurate measurement, specifically estimation of linear and volume dimensions.

Data and Methodology

Twelve complete stamped jars (SJ) have thus far been published: nine from Lachish (Zimhoni 2004; for the volume, as measured manually, see Ussishkin 2004, p. 2145), one from Tel Erani (Eitan 1973), and two from Tel Batash (Mazar and Panitz-Cohen 2001). We have directly measured seven of them, which were available to us (five from Lachish, one from Tel Erani, and one from Tel Batash). We have attained the information on the measurements of the other five jars—one or two vertical profiles—from the published scaled drawings. In what follows, we formulate our assumptions on the basis of the seven directly measured stamped jars (SJ_7), and then extend them to the entire group (SJ_{12}).

Together with the jars that carry *lmlk* seal impression, excavation reports present many storage jars of similar shape and size with unstamped handles. The latter are considered as belonging to the same group and, probably, produced in the same workshops (Mommsen *et al.* 1984, p. 106). Though these jars have limited value for our study (below), we considered them in brief. Information on the unstamped jars (UJ) was obtained from scaled drawings of one or two vertical profiles presented in the excavation reports. These include 28 items from Lachish (Zimhoni 2004), 17 from Tel Batash (Mazar and Panitz-Cohen 2001), 6 from Tel Ira (Beit-Arieh 1999), 3 from Beer Sheba (Aharoni 1973), 5 from Arad (Herzog *et al.* 1984; Singer-Avitz 2002), 2 from Gezer (Gitin 1990), 1 from Aroer (Biran and Cohen 1981), and 1 from Khirbet Rabud (Kochavi 1973); 63 vessels altogether (UJ_{63} below).

To investigate concepts of standardization and precision one needs to measure the linear dimensions, shape and volume of the storage jars (previous estimate of their volume was done by filling them with small balls of polistiren; see Ussishkin 1978, p. 76; Ussishkin 2004, p. 2145). It was difficult to take these measurements even for some of the SJ_7 group, as the items are fragile restorations. To overcome this obstacle we constructed a computer 3D model for each jar on the basis of digital pictures of its vertical profile taken at different angles, and analyzed the dimensions of the model instead of the real item. One storage jar—Lachish 8162/6(2) — was processed with the help of a 3D scanner, and the precise 3D model of its external surface (Fig. 3) was used for verifying our method.

As can be seen from Fig. 3, Lachish Storage Jar 8162/6(2), as all other jars, is somewhat asymmetric. The asymmetry is of major importance for our study; in order to capture it in full we constructed the 3D model of each item in the SJ_7 group [including Lachish Storage Jar 8162/6(2)] based on 24 profiles photographed and measured at every 15° angle. Twelve profiles were used for calculations, while the

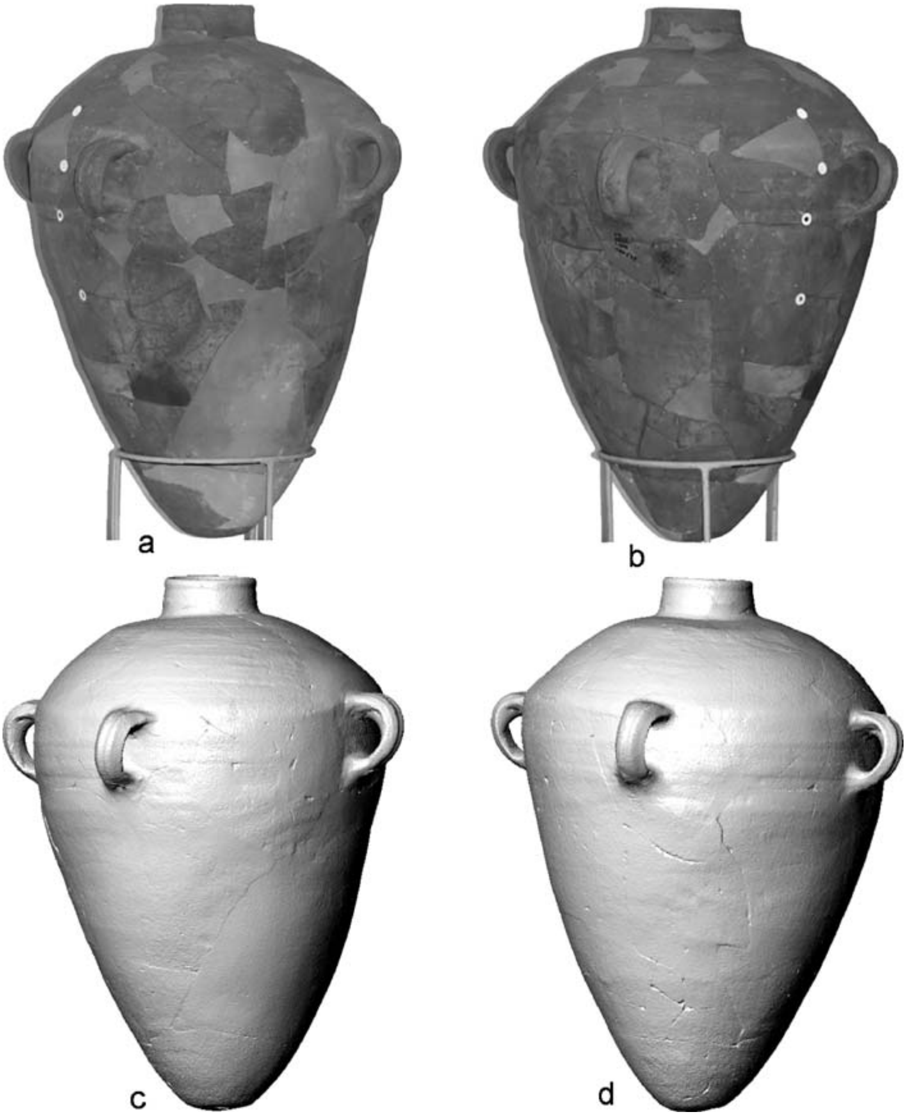

Fig. 3 Lachish Storage Jar 8162/6(2): two photos at different angles (**a, b**) and the models constructed with the 3D scanner (**c, d**) shown at the same angles.

other 12 (taken from the opposite side) were used for control. The particular camera–frame–jar installation is shown in Fig. 4 and is described in detail elsewhere (Zapassky *et al.* 2006).

Each of the 24 images of every item in the SJ_7 group was geo-referenced with MapInfo™ GIS software and then transformed into 3D with Rhinoceros™ software. The Rhinoceros™ was further employed for constructing the external surface of the jar (Fig. 5), with the vertical axes established according to the method used in our previous work (Zapassky *et al.* 2006).

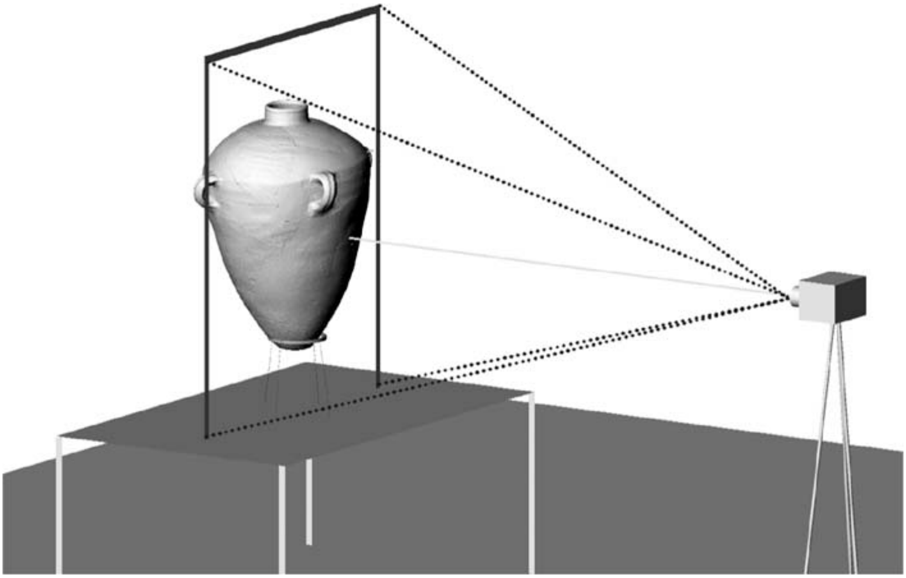

Fig. 4 Installing the storage jar and photographic equipment in order to photograph a profile. The image distortions are adjusted based on the knowledge of the width and height of the frame, the level of the camera and of the frame, and the distance between the camera and the center of the frame (see Zapassky *et al.* 2006 for details).

The 3D models for the five jars that could not be measured directly, and were therefore represented by only one or two profiles, were constructed by applying the same algorithm, evidently with lower accuracy of the results.

Estimating the Jars' Volume

Calculating the volume of a 3D model is a procedure of the Rhinoceros™, which is based on standard mathematical formulae. For the simple case of one profile, these

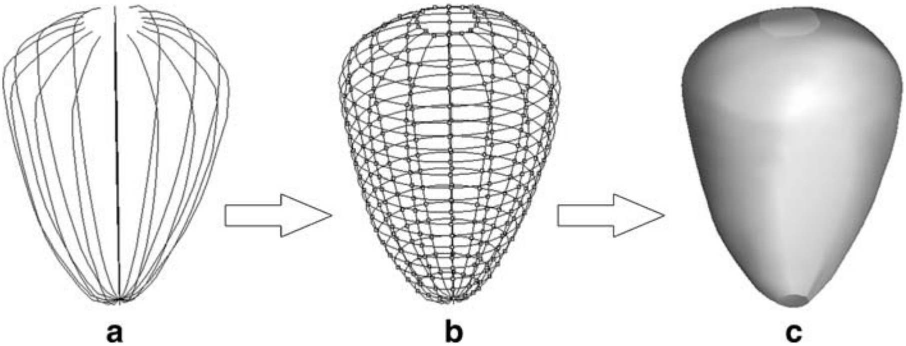

Fig. 5 The sequential stages of 3D jar modeling on the basis of multiple vertical profiles. Twelve profiles taken at every 15° enable precise reconstruction of the jar's external surface (see Zapassky *et al.* 2006 for details).

formulae have been presented in archaeological literature several times (Senior and Birnie 1995; Karasik and Smilanski 2006; Thomas and Wheeler 2008). The Rhinoceros™ enables accurate volume calculation for complex 3D solids. We employed this capability for the vessels, constructed from multiple profiles taken from different angles.

To properly calculate vessel volume, one needs to estimate the width of the vessel's wall and determine the highest possible level of the liquid in the storage jar. We measured the width of the jars' wall through ten holes between the pasted sherds of restored Lachish Storage Jar 8162/6(2) (Fig. 6a). The width of the wall at the lowest point (~10 cm from the bottom) was 1.0 cm and the typical width at the top of the vessel was 0.5 cm (top measurements differed by 0.1 cm, which is the level of the measurements' precision; Fig. 6b). We approximated the dependence of the side width on the height by hyperbolic function (Fig. 6b). In all calculations below we assumed that the liquid level in the jar reached the lower part of the neck (Fig. 6c).

We characterize the groups of jars with the mean (M), standard deviation (SD), and coefficient of variation (CV = SD/M), the latter making it possible to compare the variation of the groups of jars of different mean size.

To verify the use of 3D models instead of the real jars we compared the calculated volume of three complete jars of the SJ_7 group to their direct volume estimates achieved by pouring styrofoam into the vessels. The results and the relative error estimated as $(V_{Calculated} - V_{Measured})/V_{Measured}$ are given in Table I. As can be seen, in all three cases the error is very low.

Results

In what follows we present the results for the *lmlk* jars in biblical units. We follow the conventional wisdom that the units of length in the Iron Age were fingers and cubits, while the units of volume were *assirons* and *baths* (for these biblical units, see Powell 1992). The finger (denoted below as f) is about 1.865–1.875 cm and the cubit (denoted below as c) equals 28 fingers ~52.3–52.5 cm. As for the volume units, one *bath* (denoted below as b) is about 22.4 L=10 *assirons* (denoted below as a).

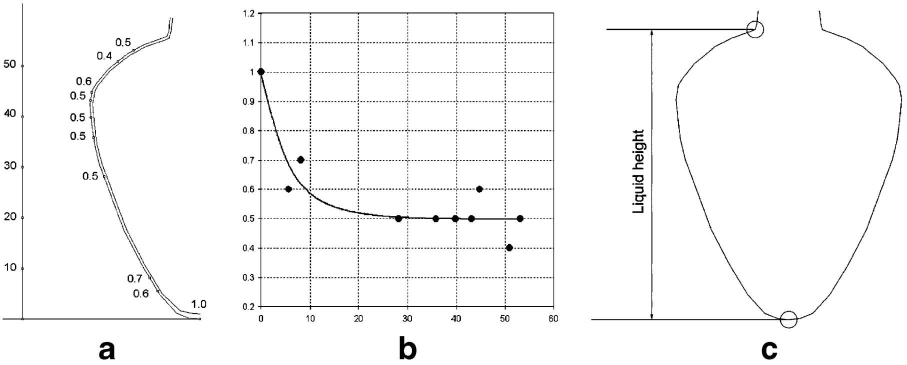

Fig. 6 A profile of the vessel with the position of the holes (**a**); graph of the wall width versus the height (**b**); level of the liquid (**c**).

Table I Accuracy of the 3D Digital Models' Volume Estimates

Lachish no.	Calculated (cm)	Measured (cm)	Relative error
75–244(1)	44.23	44.25	0.000
78–1418(2)	46.25	46.75	−0.011
8162/6(2)	39.53	39.75	−0.005

Figure 7 presents the profiles employed for constructing the 3D models of the SJ_7 vessels (each vessel is represented by 12 profiles) and their calculated volumes in *assirons*.

To fully characterize the sample, Fig. 8 presents a histogram of the volume of the SJ_7 and SJ_{12} groups. The average volume of the SJ_7 items is 20.1 a, the STD=2.1 a, and the CV=10.44%. For the entire SJ_{12} group the average volume is 20.06 a, the STD=1.56 a, and the CV=7.78%. The largest SJ_7 vessel (22.83 a) is 5.2 a (~25% of average volume) larger than the smallest one (17.63 a).

To characterize the variations in the profiles, we scaled the jars in order to give them the same height. Lachish Storage Jar 10074/1, with a volume of 19.8 a, was selected as the basis for the scaling. If we denote this jar as S, the linear dimensions of every other jar J were reduced H_J/H_S times, where H_S is the height of S and H_J of J. Since the potters could have easily controlled the height of the jars, the variability of the scaled jars reflects the ability of the potters to preserve the jars' shape. In what follows, we denote as SSJ_7 and SSJ_{12} the groups of vessels obtained with scaling from the SJ_7 and SJ_{12} jars.

For studying variations in the jars' shape we compared the maximal width of the 12 profiles of each of the SSJ_7 items (Fig. 9).

As shown in Table II, the mean maximal width of the SSJ_7 items varies between 41.0 and 44.9 cm, while the STD varies between 0.27 and 0.74 cm, and the CV between 0.6 and 1.7%. There is no correlation between the means and STD of the jars' maximal width. The overall mean maximal width of the jars is 42.9 cm, with the CV=2.98%.

The mean volume of the SSJ_7 jars equals to 19.85 a, while the variation of the volume of their 3D models is given by STD=0.68 a, with CV=3.43%. The mean volume of the entire SSJ_{12} set equals to 19.50 a, with STD=0.86 a, and CV=4.41% (Fig. 10).

As demonstrated above, based on the SSJ_7 items, the ancient potters could reproduce the shape and volume of the storage jars at 3.4% accuracy (Table II,

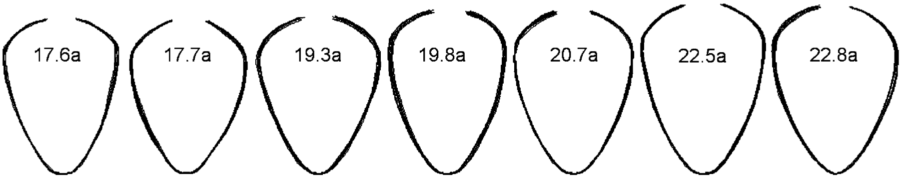

Fig. 7 Profiles and volumes (in *assirons*) of the SJ_7 group.

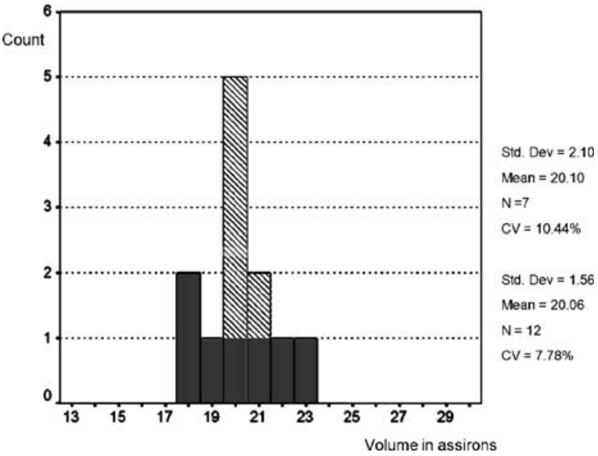

Fig. 8 Histogram of the SJ_7 and SJ_{12} volumes; in cross-hatching; jars of the SJ_{12} that do not belong to the SJ_7 group.

Fig. 10). This figure fully corresponds to Eerkens' estimates of human production perfection (Eerkens 2000). As he demonstrated, amateurs asked to reproduce a form of a simple artifact succeeded in doing so at 5% level of variation (measured by CV). A coefficient of variation equal to 2–3% indicates high individual skills or using rules, scales and measuring instruments.

The real variation of the *lmlk* SJ_7 jars' volume is, however, three times higher: 10.4% (Fig. 8). This figure is somewhat surprising as the *lmlk* jars were used in Judah's state administration, under royal auspices, for transporting valuable products. Moreover, Judah at that time was an Assyrian vassal and the *lmlk* jars could have been linked to the economic relationship with an empire. Therefore, one would expect maximal possible accuracy, which could easily be achieved by producing jars of the same height.

This raises three questions: (1) Why did the potter not aim at constructing identical jars despite an ability to do so? (2) Why preserve non-standard jars? And (3), more generally, how did the potters (plus officials and customers) know the real volume of each jar?

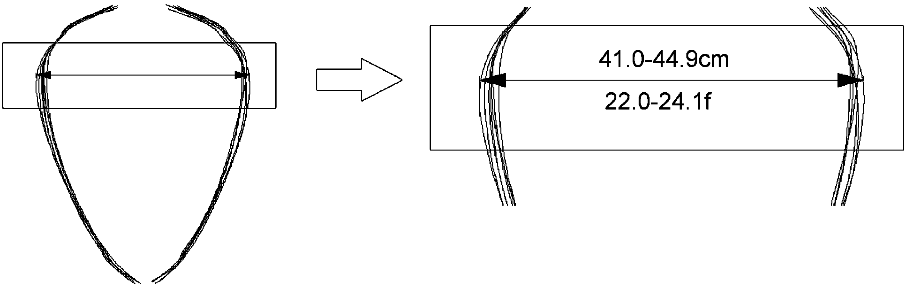

Fig. 9 Variation in average width of a jar's profiles.

Table II The Mean, SD and CV of the Maximal Width of the SSJ_7 Items

Jar no.	Mean	SD	CV
1	43.5	.63	1.45
2	43.7	.36	0.82
3	41.0	.59	1.44
4	42.3	.27	0.64
5	42.3	.43	1.02
6	42.9	.74	1.72
7	44.9	.65	1.45
Total	42.9	1.28	2.98

How was the Volume of the *lmlk* Jars Measured?

We believe that there must have been an easy and quick way to *measure* the volume of the *lmlk* jars and therefore there was no need to reproduce jars of equal volume. Theoretically, this could have been done by pouring the content of a jar into smaller, standard vessels, possibly at the gate of the city (Ephal and Naveh 1993). Yet, besides the inevitable loss of some valuable liquid, this would have been time-consuming when dealing with a large number of vessels. We therefore assume that the measuring of jars filled with liquid must have been performed on the basis of their *external dimensions*. We suggest that the Judahite state administrators applied an ‘algorithm’ for estimating the volume of the *lmlk* jars based on their external measurements. This means that the seal with the royal insignia guaranteed no more than the standard *shape* of the jar and the standard width of its wall. The state administrators could rely on the accuracy of these “hidden” parameters when applying the ‘algorithm’.

We propose that this ‘algorithm’ must have been a simple one, providing volume estimate at ca. 5% accuracy and applicable to jars of constant shape, close to two *baths* volume. As known to any modern student of school geometry, the volume V of a 3D solid of constant proportions depends on any of its linear dimension D, as $V \sim D^3$.

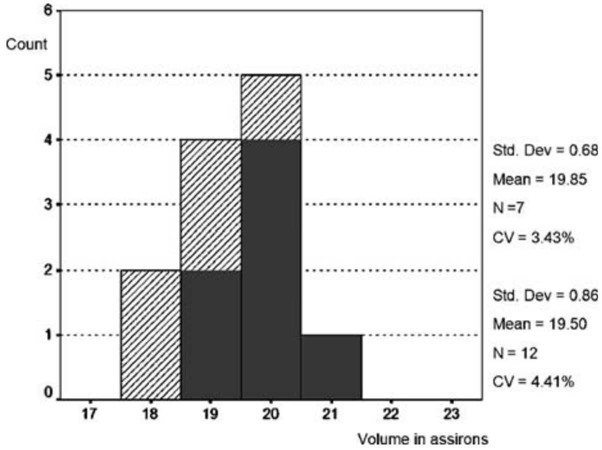

Fig. 10 Histogram of the SSJ_7 and SSJ_{12} volumes; *cross-hatching* jars of the SSJ_{12} which do not belong to the SSJ_7 group.

Yet, in search of the algorithm we limit ourselves to the simple arithmetic that could be calculated on the spot by every potter/administrator/consumer. We assume that the 'algorithm' in question might have been to take a convenient linear measurement and subtract the constant, i.e.

$$V = D - \text{const} \tag{1}$$

It is important to note that when the inherent cubic dependence is substituted by the linear one, the choice of linear dimension D and the necessary accuracy of volume estimate determine the value of constant in equation (1) and the interval of D where this formula can be applied. One should bear in mind that the V in equation (1) should be considered in *baths* or *assirons* and D in fingers or cubits—the units used in Iron Age Judah.

The question should therefore be rephrased as follows: What could have been the linear dimensions that could provide a handy, precise 'algorithm' for the *lmlk* jars? Admittedly, very few linear dimensions of the complex shape of the *lmlk* jars could be conveniently and unambiguously measured. We are able to propose only two (Fig. 11): half of the maximal horizontal circumference (P) and half of the vertical circumference (L). One can imagine these measurements performed with a string marked with finger/cubit units.

The equations of type (1) above that provide the best approximation for the SJ_7/ SJ_{12} jars volume V (*assiron*) on the basis of P (finger) or L (finger) are (see Fig. 12):

$$V(a) = P(f) - 16f \tag{2}$$

$$V(a) = L(f) - 19f \tag{3}$$

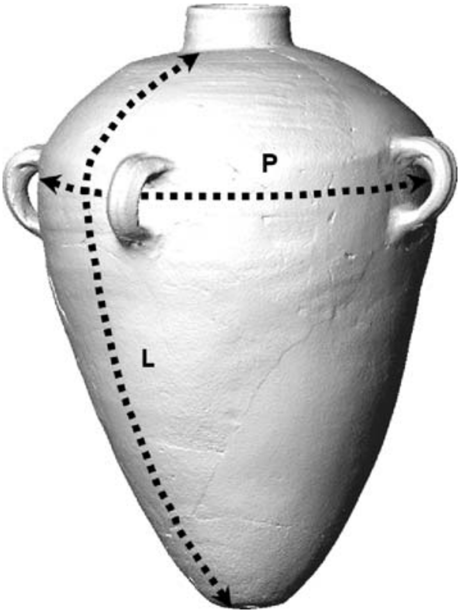

Fig. 11 Possible linear dimensions (*P* and *L*) which may have been used for a convenient 'algorithm'.

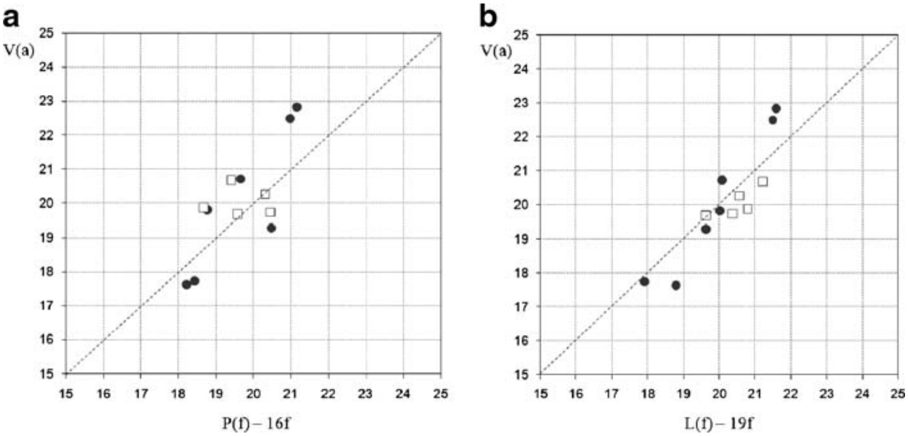

Fig. 12 The goodness of fit for the estimates of the SJ_7 and SJ_{12} *lmlk* jar's volume calculated on the basis of (**a**) half maximal horizontal circumference (*P*), and (**b**) half of the vertical circumference (*L*). Jars which belong to the SJ_7 are marked as *black circles*, the rest of the SJ_{12} jars appear as *white squares*.

Each of these formulae approximates the *lmlk* jars volume at 3–4% accuracy, the latter characteristic of their shapes. However, we failed to find a meaningful explanation for the values of constants 16f and 19f in equations (2) and (3). In addition, these formulae are "risky" for use in the sense that they can provide a wrong volume when only one of the values of P or L is characteristic of the *lmlk* jars.

At the same time, we have noticed that for a *lmlk* jar of 20 *assirons* (two *baths*) the *sum* of P and L in fingers equals 76. That is, this jar's volume in *assirons* can be expressed as:

$$V(a) = [P(f) + L(f)] - 56f$$

Now, note that 56 fingers equals two cubits. In other words, in order to estimate the volume of a *lmlk* jar within the native range of its variability (17–23 *assirons*) the ancients needed to measure P and L as shown in Fig. 13 and subtract *two cubits*:

$$V(a) = [P(f) + L(f)] - 2c \qquad (4)$$

This formula provides a good approximation of the *lmlk* jar's volume with the CV= 3.8%. The additional advantage of equation (4) over (2) and (3) is its relation to the entire shape of the jar and not only to one of its two dimensions.

Figure 14 presents the scatterplot of the UJ_{63} (unstamped) jars volume as dependent on their P(f) + L(f) − 2c. As can be expected, despite higher variations of the volume and P and L dimensions, these jars satisfy the proposed equation (4) quite well for the jars with volumes between 18 and 24 assirons and underestimate the volume for smaller jars. This confirms that the majority of these unstamped jars indeed belong to the *lmlk* group, and their volume can be measured on the basis of their outer linear dimensions, which can be conveniently taken.

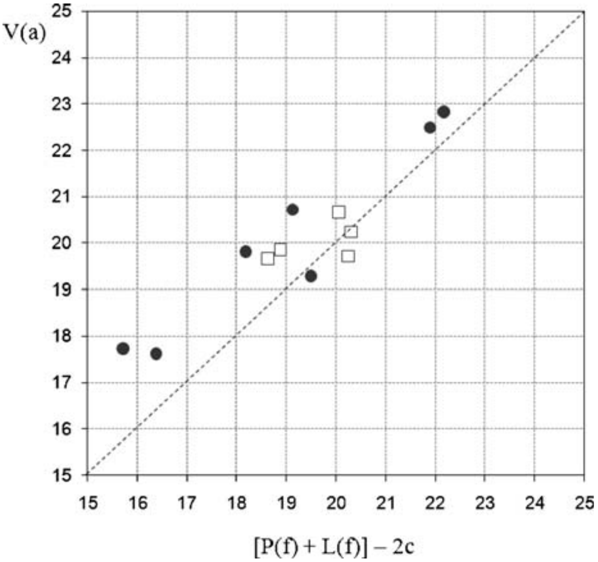

Fig. 13 The goodness of fit for V(a) = [P(f) + L(f)] − 2c estimate of the SJ_7 and SJ_{12} *lmlk* jar's volume. Jars which belong to the SJ_7 are marked as *black circles*, the rest of the SJ_{12} jars appear as *white squares*.

Conclusions

In this paper, we have investigated certain aspects of computing intelligence in antiquity by studying the ancient people's ability to reveal approximate (but inherent) geometric relationships between form and volume and to deploy them in everyday life. Our observations are based on a case-study: the volume and shape of

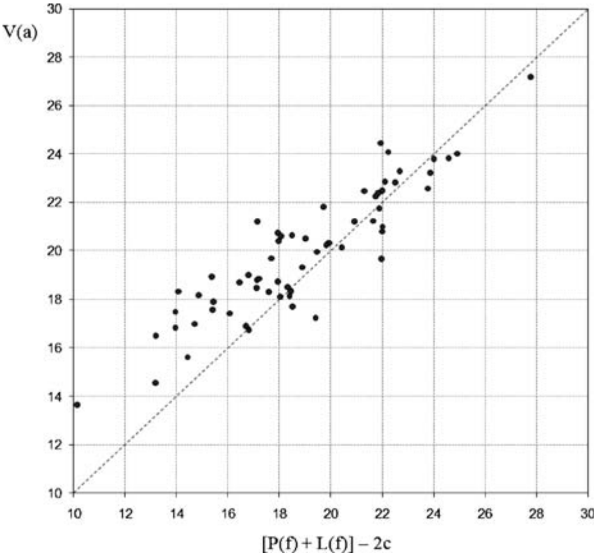

Fig. 14 The goodness of fit for V(a) = [P(f) + L(f)] − 2c estimate of the UJ_{63} jar's volume.

the *lmlk* ("belonging to the king") royal storage jars which should represent a high level of standardization in eighth century BCE Judah. To estimate the volume of these fragile items we constructed a computer 3D model for each jar. We found that the variation in the jars' volume is 7–10%, while the variation in their linear dimensions is 1–2% only; the latter is characteristic of human-made objects produced by professionals without employing measurement tools. Had the potters produced jars of the same height (a relatively easy task), the variation in their shape could have resulted in 3–4% accuracy in the volume. To explain why the potters did not do this, we hypothesize that they focused on the jars' shape and width of wall, while estimating the volume according to the jars' outer measurements. Analyzing the jars volume and shape in ancient units of length and volume, we propose two simple measurements of this kind: half of the maximal horizontal circumference (P) and half of the vertical circumference (L) (Fig. 11). We further suggest a "convenient" formula that could have been employed by the potters and customers for calculating the jars volume: volume (*assirons*) = [P (fingers) + L (fingers)] − 2 (cubits). This formula provides 3–4% accuracy and is based on the units of measurement employed in the kingdom of Judah in the Iron Age IIB, i.e. *bath*/*assiron* for measuring volume and finger/cubit for measuring length.

In more general terms, we consider our approach as an attempt to reveal everyday computational intelligence in antiquity. Textual evidence from the kingdom of Judah attests for precise quantitative formulation of wholesale invoices (see Aharoni 1981 for the Arad ostraca); yet, the volume of vessels—even the royal storage jars—varies. The question remains, then, how trade in valuable liquids, such as wine and olive oil, could be effectively managed. We hypothesize that this was done with the help of commonly-known and simple approximate rules, which—as equation (4)—enable estimating the vessels' volume on the basis of their linear dimensions.

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